Notation on the beam parameters HTE - 1.11.2016

We refer to line 9 of the Matlab file named "HypSino_SP.m", where ALarr parameter is defined as

```
ALarr = [1 0;1 1;1 1;1 -1];
```

We can convert this into the following representation

$$\text{ALarr} = 1\ 0;1\ 1;1\ 1;1\ -1 = \begin{array}{c} \begin{matrix} A_{l_1} & A_{l_2} \end{matrix} \\ \left[\begin{matrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{matrix}\right] \end{array} \begin{matrix} \leftarrow \text{Beam type 1 (Gaussian)} \\ \leftarrow \text{Beam type 2 (cos or cosh Gaussian)} \\ \leftarrow \text{Beam type 3 (cos or cosh Gaussian)} \\ \leftarrow \text{Beam type 4 (sine or sinh or annular Gaussian)} \end{matrix} \qquad (1)$$

To make an association, we take the definitions of lowest and Higher Order Sinusoidal Hyperbolic Gaussian Beams from the lecture notes entitled, "ECE 635_Free space propagation notes_Eylul 2011_HTE" respectively in cylindrical and Cartesian coordinates, i.e. (1.4a) and (1.4b)

$$\begin{aligned} u_s(s,\phi_s) &= \sum_{\ell=1}^{N} A_\ell \exp\left[-k\alpha_\ell s^2 + (\sin\phi_s + \cos\phi_s) D_{s\ell} s\right] \\ &= \sum_{\ell=1}^{2} A_\ell \exp\left[-k\alpha_\ell s^2 + (\sin\phi_s + \cos\phi_s) D_{s\ell} s\right] \\ &= A_{\ell_1} \exp\left[-k\alpha_{\ell_1} s^2 + (\sin\phi_s + \cos\phi_s) D_{s\ell_1} s\right] + A_{\ell_2} \exp\left[-k\alpha_{\ell_2} s^2 + (\sin\phi_s + \cos\phi_s) D_{s\ell_2} s\right] \\ u_s(s_x, s_y) &= \sum_{\ell=1}^{N} A_\ell H_{n_\ell}(a_{x\ell} s_x + b_{x\ell}) \exp\left[-\left(0.5k\alpha_{x\ell} s_x^2 - D_{x\ell} s_x\right)\right] \\ &\quad \times H_{m_\ell}(a_{y\ell} s_y + b_{y\ell}) \exp\left[-\left(0.5k\alpha_{y\ell} s_y^2 - D_{y\ell} s_y\right)\right] \\ &= \sum_{\ell=1}^{2} A_\ell H_{n_\ell}(a_{x\ell} s_x + b_{x\ell}) \exp\left[-\left(0.5k\alpha_{x\ell} s_x^2 - D_{x\ell} s_x\right)\right] \\ &\quad \times H_{m_\ell}(a_{y\ell} s_y + b_{y\ell}) \exp\left[-\left(0.5k\alpha_{y\ell} s_y^2 - D_{y\ell} s_y\right)\right] \\ &= A_{\ell_1} H_{n_{\ell_1}}(a_{x\ell_1} s_x + b_{x\ell_1}) \exp\left[-\left(0.5k\alpha_{x\ell_1} s_x^2 - D_{x\ell_1} s_x\right)\right] H_{m_{\ell_1}}(a_{y\ell_1} s_y + b_{y\ell_1}) \exp\left[-\left(0.5k\alpha_{y\ell_1} s_y^2 - D_{y\ell_1} s_y\right)\right] \\ &+ A_{\ell_2} H_{n_{\ell_2}}(a_{x\ell_2} s_x + b_{x\ell_2}) \exp\left[-\left(0.5k\alpha_{x\ell_2} s_x^2 - D_{x\ell_2} s_x\right)\right] H_{m_{\ell_2}}(a_{y\ell_2} s_y + b_{y\ell_2}) \exp\left[-\left(0.5k\alpha_{y\ell_2} s_y^2 - D_{y\ell_2} s_y\right)\right] \end{aligned} \qquad (2)$$

As seen from (2) all other parameters such as $n_l,\ a_{x\ell},\ b_{x\ell},\ D_{x\ell},\ m_l,\ a_{y\ell},\ b_{y\ell},\ D_{y\ell}$ have to be supplied similar to ALarr shown in (1). This way we write the numeric values of $n_l,\ a_{x\ell},\ b_{x\ell},\ D_{x\ell},\ m_l,\ a_{y\ell},\ b_{y\ell},\ D_{y\ell}$ in the form of a matrix of 4 X 2.

Additional requirements for the beam types

α_{sl}

alfasarr = 1e-2 1e-2;1e-2 1e-2;1e-2 1e-2;1e2 0.95e-2

$$= \begin{bmatrix} \alpha_{s1} & \alpha_{s2} \\ \text{1e-2} & \text{1e-2} \\ \text{1e-2} & \text{1e-2} \\ \text{1e-2} & \text{1e-2} \\ \text{1e-2} & \text{0.95e-2} \end{bmatrix} \begin{matrix} \\ \leftarrow \text{Beam type 1 (Gaussian)} \\ \leftarrow \text{Beam type 2 (cos, cosh, sine, sinh Gaussian)} \\ \leftarrow \text{Beam type 3 (cos, cosh, sine, sinh Gaussian)} \\ \leftarrow \text{Beam type 4 (annular Gaussian)} \end{matrix} \quad (3)$$

Dmat = [0 25 50 200 200 250];

D_{sl}

Darr = [1*Dmat(1) -1*Dmat(1);1*Dmat(4) -1*Dmat(4);j*Dmat(4) -j*Dmat(4);
j*Dmat(1) -j*Dmat(1)];

$$= \begin{bmatrix} D_{sl_1} & D_{sl_2} \\ \text{Dmat(1)} & -\text{Dmat(1)} \\ \text{Dmat(4)} & -\text{Dmat(4)} \\ j\text{Dmat(4)} & -j\text{Dmat(4)} \\ j\text{Dmat(1)} & -j\text{Dmat(1)} \end{bmatrix} \begin{matrix} \\ \leftarrow \text{Beam type 1 (Gaussian)} \\ \leftarrow \text{Beam type 2 (cosh Gaussian) if ALarr = [1 1]} \\ \leftarrow \text{Beam type 3 (cos Gaussian) if ALarr = [1 1]} \\ \leftarrow \text{Beam type 4 (annular Gaussian)} \end{matrix} \quad (4)$$

Dmat = [0 25 50 200 200 250];

D_{sl}

Darr = [1*Dmat(1) -1*Dmat(1);1*Dmat(4) -1*Dmat(4);j*Dmat(4) -j*Dmat(4);
j*Dmat(1) -j*Dmat(1)];

$$= \begin{bmatrix} D_{sl_1} & D_{sl_2} \\ \text{Dmat(1)} & -\text{Dmat(1)} \\ \text{Dmat(4)} & -\text{Dmat(4)} \\ j\text{Dmat(4)} & -j\text{Dmat(4)} \\ j\text{Dmat(1)} & -j\text{Dmat(1)} \end{bmatrix} \begin{matrix} \\ \leftarrow \text{Beam type 1 (Gaussian)} \\ \leftarrow \text{Beam type 2 (sinh Gaussian) if ALarr = [1 -1]} \\ \leftarrow \text{Beam type 3 (sine Gaussian) if ALarr = [j -j]} \\ \leftarrow \text{Beam type 4 (annular Gaussian)} \end{matrix} \quad (5)$$

Exercise_SH : Reorganize HypSino_SP.m so that you can obtain, sine and sinh Gaussian beams. Make your plots and paste them in the Exercise folder.