Çankaya University – ECE Department – ECE 635 (FE)

Student Name : Date : 10.01.2017

Student Number : Open book exam

Questions

1. (70 Points) By using the MATLAB m file "ParCoh_SinoHypR.m" from ece635 course webpage or by using (5.17) of lecture note entitled, "ECE 635_Free space propagation notes Eylul 2011_HTE" for a propagation distance of L, give (only the relevant) parameter settings and sample numeric values of

   a. Fully coherent Gaussian beam.

   b. Partially coherent Gaussian beam.

   c. Partially coherent cosh and sinh Gaussian beams.

   d. Fully coherent cos and sine Gaussian beams.

   e. Partially coherent annular Gaussian beam.

Modify the mutual coherence function expression given in (5.17) of lecture notes entitled, "ECE 635_Free space propagation notes Eylul 2011_HTE" so that it turns into the intensity expressions of above beams.

Solution : All options of a to e are listed below

a. Fully coherent Gaussian beam

ALarr $\rightarrow [A_1 \; A_2] = [1 \; 0]$; alfasxarr $\rightarrow [\alpha_{sx1} \; \alpha_{sx21}] = $ [1e-2 1e-2] ; alfasyarr = alfasxarr

Dxarr $\rightarrow [D_{x1} \; D_{x2}] = [0 \; 0]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2 =$ 1e15

b. Partially coherent Gaussian beam

ALarr $\rightarrow [A_1 \; A_2] = [1 \; 0]$; alfasxarr $\rightarrow [\alpha_{sx1} \; \alpha_{sx21}] = $ [1e-2 1e-2] ; alfasyarr = alfasxarr

Dxarr $\rightarrow [D_{x1} \; D_{x2}] = [0 \; 0]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2 =$ 1e-4

c1. Partially coherent cosh Gaussian beam

ALarr $\rightarrow [A_1 \; A_2] = [1 \; 1]$; alfasxarr $\rightarrow [\alpha_{sx1} \; \alpha_{sx21}] = $ [1e-2 1e-2] ; alfasyarr = alfasxarr

Dxarr $\rightarrow [D_{x1} \; D_{x2}] = [100 \; -100]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2 =$ 1e-4

c2. Partially coherent sinh Gaussian beam

ALarr $\rightarrow [A_1 \; A_2] = [1 \; -1]$; alfasxarr $\rightarrow [\alpha_{sx1} \; \alpha_{sx21}] = $ [1e-2 1e-2] ; alfasyarr = alfasxarr

Dxarr $\rightarrow [D_{x1} \; D_{x2}] = [100 \; -100]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2 =$ 1e-4 d1. Fully coherent cos Gaussian beam

ALarr $\rightarrow \left[A_1 \; A_2 \right] = [1 \; 1]$; alfasxarr $\rightarrow \left[\alpha_{sx1} \;\; \alpha_{sx21} \right] = [\text{1e-2 1e-2}]$; alfasyarr = alfasxarr

Dxarr $\rightarrow \left[D_{x1} \; D_{x2} \right] = [j100 \; - j100]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2$ = 1e15

d2. Fully coherent sine Gaussian beam

ALarr $\rightarrow \left[A_1 \; A_2 \right] = [j \;\; -j]$; alfasxarr $\rightarrow \left[\alpha_{sx1} \;\; \alpha_{sx21} \right] = [\text{1e-2 1e-2}]$; alfasyarr = alfasxarr

Dxarr $\rightarrow \left[D_{x1} \; D_{x2} \right] = [j100 \; - j100]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2$ = 1e15

e. Partially coherent annular Gaussian beam

ALarr $\rightarrow \left[A_1 \; A_2 \right] = [1 \;\; -1]$; alfasxarr $\rightarrow \left[\alpha_{sx1} \;\; \alpha_{sx21} \right] = [\text{1e-2 0.9e-2}]$; alfasyarr = alfasxarr

Dxarr $\rightarrow \left[D_{x1} \; D_{x2} \right] = [0 \;\; 0]$; Dyarr = Dxarr ; sig2 $\rightarrow \sigma_s^2$ = 1e-4

The steps of modification of the mutual coherence function expression given in (5.17) of lecture notes entitled, “ECE 635_Free space propagation notes Eylul 2011_HTE” to deliver the intensity expressions of above beams are listed below.

Mutual coherence function expression given in (5.17) of lecture notes entitled, "ECE 635_Free space propagation notes Eylul 2011_HTE"

$$\Gamma_r\left(r_{1x}, r_{1y}, r_{2x}, r_{2y}, L\right) = k\sigma_s^2 \exp\left[j\frac{k}{2L}\left(r_{1x}^2 - r_{2x}^2 + r_{1y}^2 - r_{2y}^2\right)\right] \sum_{\ell_1=1}^{N}\sum_{\ell_2=1}^{N}$$

$$\frac{A_{\ell_1} A_{\ell_1}^*}{\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{x\ell_1} + \alpha_{x\ell_2}^*\right)L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]^{0.5}\left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{y\ell_1} + \alpha_{y\ell_2}^*\right)L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]^{0.5}}$$

$$\exp\left(\frac{0.5}{L\left(k\alpha_{x\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)}\left\{\left(-jkr_{1x} + D_{x\ell_1}L\right)^2\sigma_s^2 + \frac{\left[-j\left(r_{1x} - r_{2x}\right)kL + j\left(\alpha_{x\ell_1}L - j\right)k^2 r_{2x}\sigma_s^2 + D_{x\ell_1}L^2 + \left(k\alpha_{x\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)D_{x\ell_2}^* L\right]^2}{k\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{x\ell_1} + \alpha_{x\ell_2}^*\right)L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]}\right\}\right)$$

$$\exp\left(\frac{0.5}{L\left(k\alpha_{y\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)}\left\{\left(-jkr_{1y} + D_{y\ell_1}L\right)^2\sigma_s^2 + \frac{\left[-j\left(r_{1y} - r_{2y}\right)kL + j\left(\alpha_{y\ell_1}L - j\right)k^2 r_{2y}\sigma_s^2 + D_{y\ell_1}L^2 + \left(k\alpha_{y\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)D_{y\ell_2}^* L\right]^2}{k\left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{y\ell_1} + \alpha_{y\ell_2}^*\right)L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]}\right\}\right) \quad (1.1)$$

Upon setting $r_{1x} = r_{2x} = r_x, r_{1y} = r_{2y} = r_y$ in (1.1), we get the intensity expression of the sinusoidal hyperbolic Gaussian beam as

$$I_r\left(r_x,r_y\right)=\Gamma_r\left(r_{1x}=r_{2x}=r_x,r_{1y}=r_{2y}=r_y,L\right)=k\sigma_s^2\sum_{\ell_1=1}^{N}\sum_{\ell_2=1}^{N}$$

$$\frac{A_{\ell_1}A_{\ell_1}^*}{\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2L^2+\left(\alpha_{x\ell_1}+\alpha_{x\ell_2}^*\right)L^2+j\left(\alpha_{x\ell_1}-\alpha_{x\ell_2}^*\right)k\sigma_s^2L+k\sigma_s^2\right]^{0.5}\left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2L^2+\left(\alpha_{y\ell_1}+\alpha_{y\ell_2}^*\right)L^2+j\left(\alpha_{y\ell_1}-\alpha_{y\ell_2}^*\right)k\sigma_s^2L+k\sigma_s^2\right]^{0.5}}$$

$$\exp\left(\frac{0.5}{L\left(k\alpha_{x\ell_1}\sigma_s^2L+L-jk\sigma_s^2\right)}\left\{\left(-jkr_x+D_{x\ell_1}L\right)^2\sigma_s^2+\frac{\left[j\left(\alpha_{x\ell_1}L-j\right)k^2r_x\sigma_s^2+D_{x\ell_1}L^2+\left(k\alpha_{x\ell_1}\sigma_s^2L+L-jk\sigma_s^2\right)D_{x\ell_2}^*L\right]^2}{k\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2L^2+\left(\alpha_{x\ell_1}+\alpha_{x\ell_2}^*\right)L^2+j\left(\alpha_{x\ell_1}-\alpha_{x\ell_2}^*\right)k\sigma_s^2L+k\sigma_s^2\right]}\right\}\right)$$

$$\exp\left(\frac{0.5}{L\left(k\alpha_{y\ell_1}\sigma_s^2L+L-jk\sigma_s^2\right)}\left\{\left(-jkr_y+D_{y\ell_1}L\right)^2\sigma_s^2+\frac{\left[j\left(\alpha_{y\ell_1}L-j\right)k^2r_y\sigma_s^2+D_{y\ell_1}L^2+\left(k\alpha_{y\ell_1}\sigma_s^2L+L-jk\sigma_s^2\right)D_{y\ell_2}^*L\right]^2}{k\left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2L^2+\left(\alpha_{y\ell_1}+\alpha_{y\ell_2}^*\right)L^2+j\left(\alpha_{y\ell_1}-\alpha_{y\ell_2}^*\right)k\sigma_s^2L+k\sigma_s^2\right]}\right\}\right) \quad (1.2)$$

Next to obtain the intensity expression of the fully coherent Gaussian beam we let $\sigma_s\to\infty,\ N=1,\ D_{x\ell_1},\ D_{x\ell_2},D_{y\ell_1},\ D_{y\ell_2}\to 0$

$$I_r\left(r_x,r_y\right)=\frac{\left|A\right|^2}{\left[\left|\alpha_x\right|^2L^2+j\left(\alpha_x-\alpha_x^*\right)L+1\right]^{0.5}\left[\left|\alpha_y\right|^2L^2+j\left(\alpha_y-\alpha_y^*\right)L+1\right]^{0.5}}\exp\left[\frac{-0.5k\left(\alpha_x+\alpha_x^*\right)r_x^2}{\left|\alpha_x\right|^2L^2+j\left(\alpha_x-\alpha_x^*\right)L+1}\right]\exp\left[\frac{-0.5k\left(\alpha_y+\alpha_y^*\right)r_y^2}{\left|\alpha_y\right|^2L^2+j\left(\alpha_y-\alpha_y^*\right)L+1}\right] \quad (1.3)$$

If we keep σ_s finite, then we obtain the intensity expression of the partially coherent Gaussian beam

$$I_r\left(r_x,r_y\right)=k\sigma_s^2\frac{\left|A_\ell\right|^2}{\left[k\left|\alpha_x\right|^2\sigma_s^2L^2+\left(\alpha_x+\alpha_x^*\right)L^2+j\left(\alpha_x-\alpha_x^*\right)k\sigma_s^2L+k\sigma_s^2\right]^{0.5}\left[k\left|\alpha_y\right|^2\sigma_s^2L^2+\left(\alpha_y+\alpha_y^*\right)L^2+j\left(\alpha_y-\alpha_y^*\right)k\sigma_s^2L+k\sigma_s^2\right]^{0.5}}$$
$$\exp\left(\frac{-0.5k^2\sigma_s^2r_x^2}{L\left(k\alpha_x\sigma_s^2L+L-jk\sigma_s^2\right)}\left\{\frac{k^2\left|\alpha_x\right|^2\sigma_s^2L^2+k\left(\alpha_x+\alpha_x^*\right)L^2+j\left(\alpha_x-\alpha_x^*\right)k^2\sigma_s^2L+k^2\alpha_x^2\sigma_s^2L^2-2j\alpha_xk^2\sigma_s^2L}{k\left[k\left|\alpha_x\right|^2\sigma_s^2L^2+\left(\alpha_x+\alpha_x^*\right)L^2+j\left(\alpha_x-\alpha_x^*\right)k\sigma_s^2L+k\sigma_s^2\right]}\right\}\right)$$
$$\exp\left(\frac{0.5k^2\sigma_s^2r_y^2}{L\left(k\alpha_y\sigma_s^2L+L-jk\sigma_s^2\right)}\left\{\frac{k^2\left|\alpha_y\right|^2\sigma_s^2L^2+k\left(\alpha_y+\alpha_y^*\right)L^2+j\left(\alpha_y-\alpha_y^*\right)k^2\sigma_s^2L+k^2\alpha_y^2\sigma_s^2L^2-2j\alpha_yk^2\sigma_s^2L}{k\left[k\left|\alpha_y\right|^2\sigma_s^2L^2+\left(\alpha_y+\alpha_y^*\right)L^2+j\left(\alpha_x-\alpha_x^*\right)k\sigma_s^2L+k\sigma_s^2\right]}\right\}\right) \quad (1.4)$$

(1.4) can further be simplified as shown in PC-Gaussian beam (located in the Vortex beams directory)

For partially coherent annular, cosh and sinh Gaussian beams (1.2) applies. Fully coherent cos and sine Gaussian beams, we will have the following intensity expression

$$
\begin{aligned}
I_r\left(r_x, r_y\right) &= \Gamma_r\left(r_{1x} = r_{2x} = r_x, r_{1y} = r_{2y} = r_y, L\right) = \sum_{\ell_1=1}^{N} \sum_{\ell_2=1}^{N} \\
&\frac{A_{\ell_1} A_{\ell_1}^*}{\left[\alpha_{x\ell_1}\alpha_{x\ell_2}^* L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)L + 1\right]^{0.5}\left[\alpha_{y\ell_1}\alpha_{y\ell_2}^* L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)L + 1\right]^{0.5}} \\
&\exp\left(\frac{0.5}{kL\left(\alpha_{x\ell_1}L - j\right)}\left\{\left(-jkr_x + D_{x\ell_1}L\right)^2 + \frac{\left[j\left(\alpha_{x\ell_1}L - j\right)kr_x + \left(\alpha_{x\ell_1}L - j\right)D_{x\ell_2}^* L\right]^2}{\left[\alpha_{x\ell_1}\alpha_{x\ell_2}^* L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)L + 1\right]}\right\}\right) \\
&\exp\left(\frac{0.5}{kL\left(\alpha_{y\ell_1}L - j\right)}\left\{\left(-jkr_y + D_{y\ell_1}L\right)^2 + \frac{\left[j\left(\alpha_{y\ell_1}L - j\right)kr_y + \left(\alpha_{y\ell_1}L - j\right)D_{y\ell_2}^* L\right]^2}{\left[\alpha_{y\ell_1}\alpha_{y\ell_2}^* L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)L + 1\right]}\right\}\right)
\end{aligned}
\quad (1.5)
$$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) Mutual coherence function refers to the intensity of the beam : True as shown in (1.2) above. It is worth pointing out that mutual coherence function can be complex in general, whereas intensity is purely real.

b) The phase of a fully coherent beam changes rapidly along time or spatial axis : False, this statement applies to partially coherent beam, particularly partially coherent beam closer to full incoherence.

c) ABCD formalism is the same as Huygens Fresnel integral : True as shown in sections 3 and 4 of lecture notes entitled, "ECE 635_Free space propagation notes_Eylul 2011_HTE". It is to be kept in mind that ABCD formalism is able to account for optical elements on the path of propagation.

d) After propagation, sine Gaussian beam turns into sinh Gaussian beam : True as verified in the runs of ParCoh_SinoHypR.m.

e) Paraxial wave approximation (PWE) applies on the source plane : False, for PWE to be applicable, the beam must have propagated.