Çankaya University – ECE Department – ECE 635 (MT)

Student Name : Date : 22.11.2016

Student Number : Open book exam

Questions

1. (35 Points) A fibre with $n_1 = 1.50,\ n_2 = 1.49,\ a = 3.8\ \mu\text{m}$ is given. Find at which wavelengths, this fibre should be operated, so that the single mode condition is satisfied. By choosing three such different wavelengths, determine the propagation constants of corresponding HE_{11} modes. For evaluations, you can use the m file Findingbeta_Exp0.m, available on the course webpage, ece635.cankaya.edu.tr.

Solution : To satisfy the condition single mode, we must have the normalized frequency of the fibre, $V < 2.405$. On the other hand, using the relationship between normalized and other parameters of the fibre, we have

$$\lambda \geq \frac{2\pi a}{V}\left(n_1^2 - n_2^2\right)^{0.5} = 1.7167\ \mu\text{m} \tag{1.1}$$

Hence select wavelengths of $\lambda_a = 1.72\ \mu\text{m},\ \lambda_b = 1.75\ \mu\text{m},\ \lambda_c = 1.78\ \mu\text{m}$, the respective normalized frequencies become

$$V_a = \frac{2\pi a}{\lambda_a}\left(n_1^2 - n_2^2\right)^{0.5} = 2.4003,\ V_b = \frac{2\pi a}{\lambda_b}\left(n_1^2 - n_2^2\right)^{0.5} = 2.3592,\ V_c = \frac{2\pi a}{\lambda_c}\left(n_1^2 - n_2^2\right)^{0.5} = 2.3194 \tag{1.2}$$

The related plots can be found in Figs. 1.1, 1.2 and 1.3.

By reading u_n and w_n values from the closest normalized frequencies given in (1.2), we evaluate propagation constants as follows

$$\text{At } \lambda_a = 1.72\ \mu\text{m},\ V_a = 2.4003\ :\ u_{n_a} = 1.645 \text{ and } w_{n_a} = 1.75$$

$$k_{1_a} = n_1 \frac{2\pi}{\lambda_a} = 5.4795\times10^6,\ k_{2_a} = n_2 \frac{2\pi}{\lambda_a} = 5.443\times10^6$$

$$\beta_{n_a} = \left(a^2 \overbrace{n_1^2 \frac{4\pi^2}{\lambda_a^2}}^{k_{1_a}^2} - u_{n_a}^2 \right)^{0.5} = 20.7571 \quad \text{or} \quad \beta_{n_a} = \left(a^2 \overbrace{n_2^2 \frac{4\pi^2}{\lambda_a^2}}^{k_{2_a}^2} + w_{n_a}^2 \right)^{0.5} = 20.7373$$

$$\beta_a = \frac{\beta_{n_a}}{a} = 5.4624\times10^6 \text{ hence } k_{2_a} < \beta_a < k_{1_a} \tag{1.3}$$

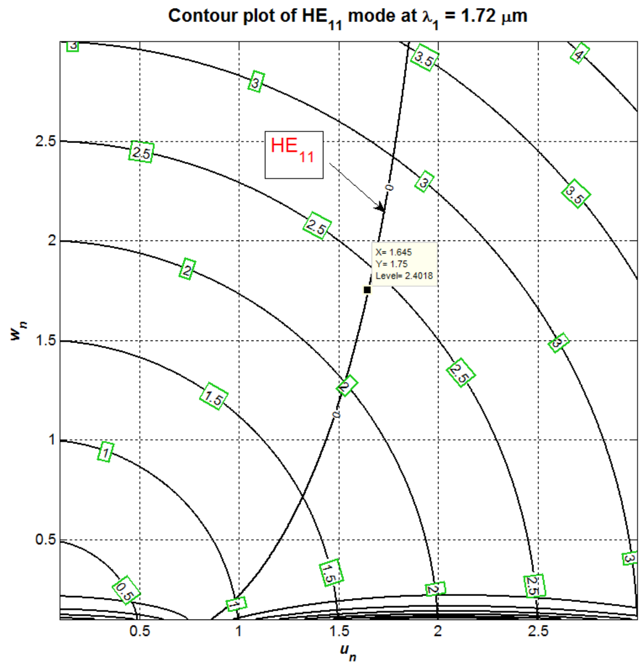

Fig. 1.1 The contour plot of HE_{11} mode at $\lambda_1 = 1.72\ \mu\mathrm{m}$.

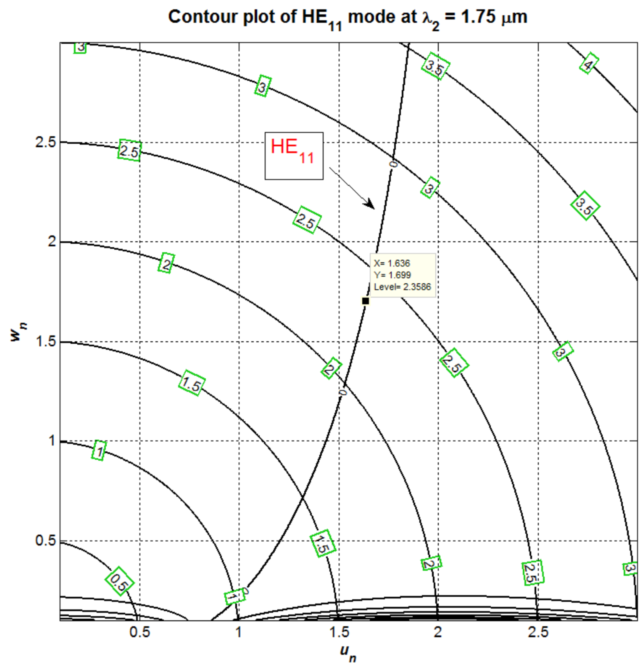

Fig. 1.2 The contour plot of HE_{11} mode at $\lambda_2 = 1.75\ \mu\mathrm{m}$.

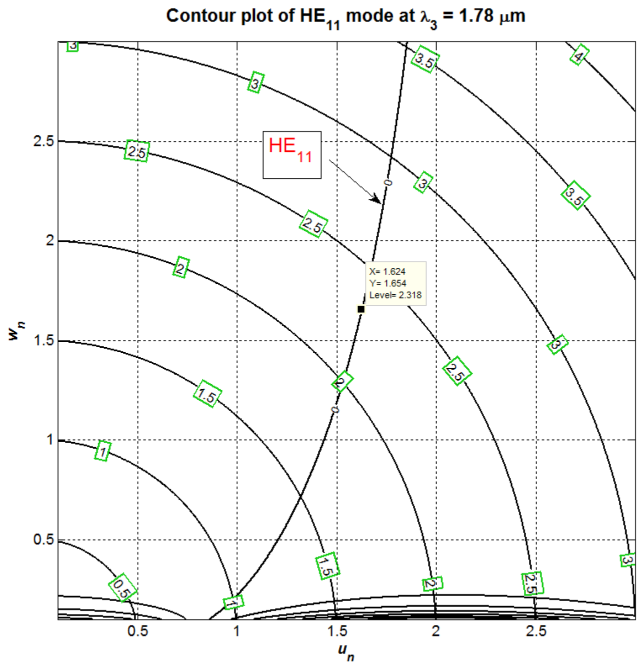

Fig. 1.3 The contour plot of HE_{11} mode at $\lambda_3 = 1.78\ \mu\mathrm{m}$.

At $\lambda_b = 1.75\ \mu\mathrm{m}$, $V_b = 2.3592\ :\ u_{n_b} = 1.636$ and $w_{n_b} = 1.699$

$$k_{1_b} = n_1 \frac{2\pi}{\lambda_b} = 5.3856\times10^6,\ k_{2_b} = n_2 \frac{2\pi}{\lambda_b} = 5.3497\times10^6$$

$$\beta_{n_b} = \left(a^2 \overbrace{n_1^2 \frac{4\pi^2}{\lambda_b^2}}^{k_{1_b}^2} - u_{n_b}^2 \right)^{0.5} = 20.3997 \quad \text{or} \quad \beta_{n_b} = \left(a^2 \overbrace{n_2^2 \frac{4\pi^2}{\lambda_b^2}}^{k_{2_b}^2} + w_{n_b}^2 \right)^{0.5} = 20.3997$$

$$\beta_b = \frac{\beta_{n_b}}{a} = 5.3684\times10^6 \text{ hence } k_{2_b} < \beta_b < k_{1_b} \tag{1.5}$$

At $\lambda_c = 1.78\ \mu\mathrm{m}$, $V_c = 2.3194\ :\ u_{n_c} = 1.624$ and $w_{n_c} = 1.654$

$$k_{1_c} = n_1 \frac{2\pi}{\lambda_c} = 5.2948\times10^6,\ k_{2_c} = n_2 \frac{2\pi}{\lambda_c} = 5.2595\times10^6$$

$$\beta_{n_c} = \left(a^2 \overbrace{n_1^2 \frac{4\pi^2}{\lambda_c^2}}^{k_{1_c}^2} - u_{n_c}^2 \right)^{0.5} = 20.0547 \quad \text{or} \quad \beta_{n_c} = \left(a^2 \overbrace{n_2^2 \frac{4\pi^2}{\lambda_b^2}}^{k_{2_c}^2} + w_{n_c}^2 \right)^{0.5} = 20.0545$$

$$\beta_c = \frac{\beta_{n_c}}{a} = 5.2775\times10^6 \text{ hence } k_{2_c} < \beta_c < k_{1_c} \tag{1.6}$$

2. (35 Points) A quadratic graded index fibre has $n_1 = 1.50$, $n_2 = 1.485$, $a = 12\ \mu$m and it is operated at $\lambda = 1.55\ \mu\text{m}$. By using the Matlab file, WKBcurves.m, available on the ECE 635 course webpage (under the heading of Lecture Notes), and changing the m and ν (denoted as L in the m file itself) values find and plot at least one sample of

A. Propagating Meridional Ray
B. Propagating Skew Ray
C. Refracting Meridional Ray
D. Refracting Skew Ray

indicating $r_{\min}$ and $r_{\max}$ points for each case. Again by using the same m file, evaluate the propagation constant for each ray above.

Solution : From the numeric values given in the question, we find

$$k_1 = n_1 \frac{2\pi}{\lambda} = 6.0805 \times 10^6,\ k_2 = n_2 \frac{2\pi}{\lambda} = 6.0197 \times 10^6 \qquad (2.1)$$

Then by inserting the given numeric values, we will have the following plots

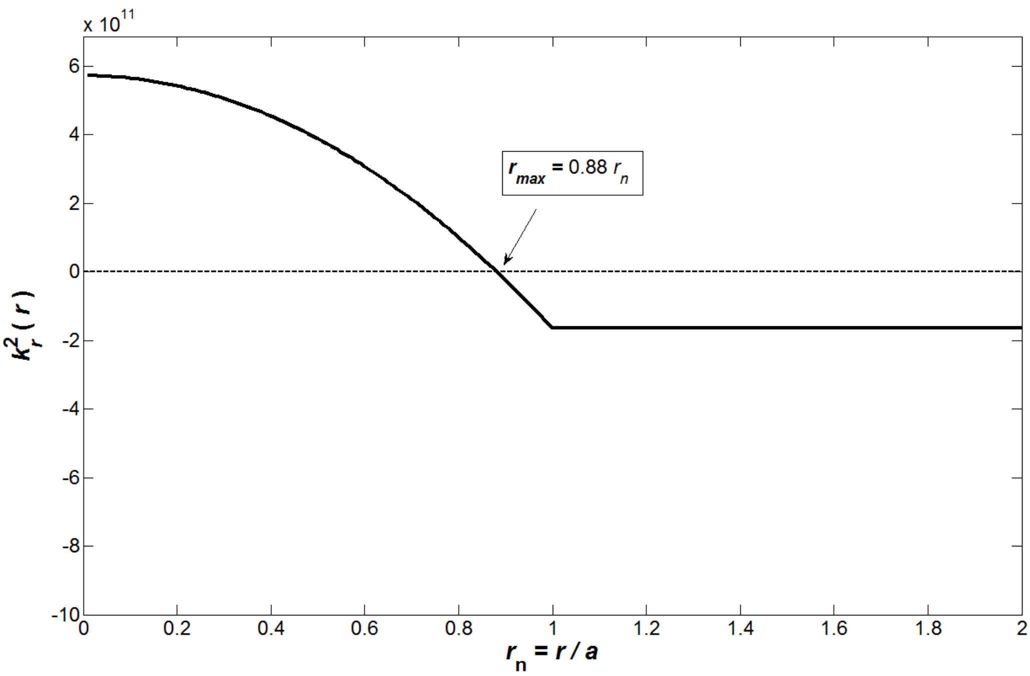

Fig. 2.1 A sample of Propagating Meridional Ray at $m = 2$ and $\nu = 0$ with $\beta = 6.0333 \times 10^6$.

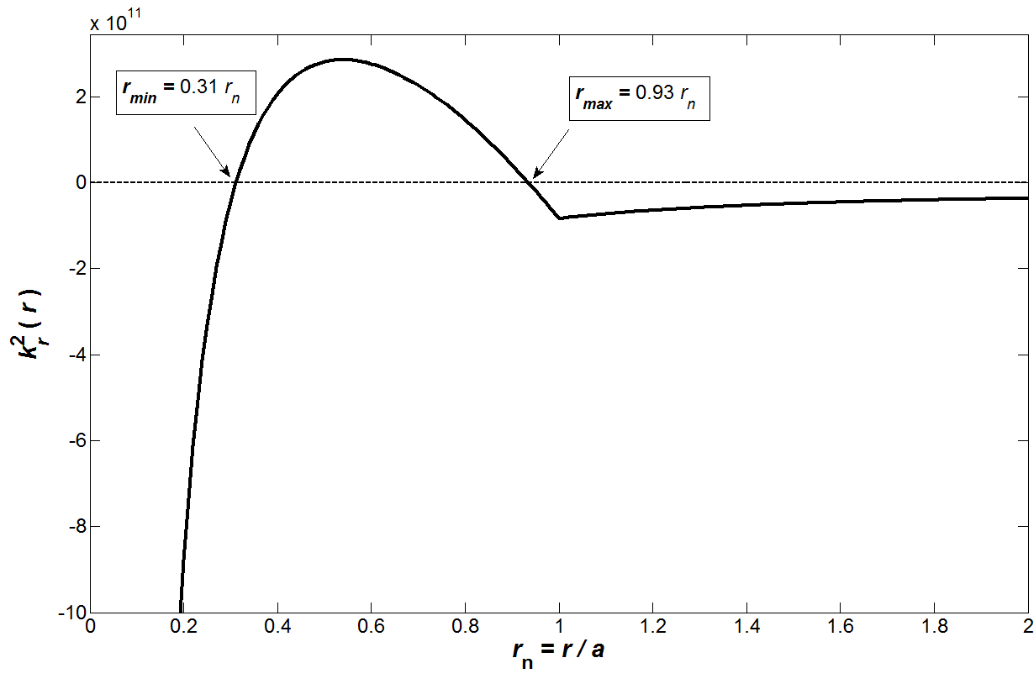

Fig. 2.2 A sample of Propagating Skew Ray at $m=1$ and $\nu=3$ with $\beta=6.0214\times10^6$.

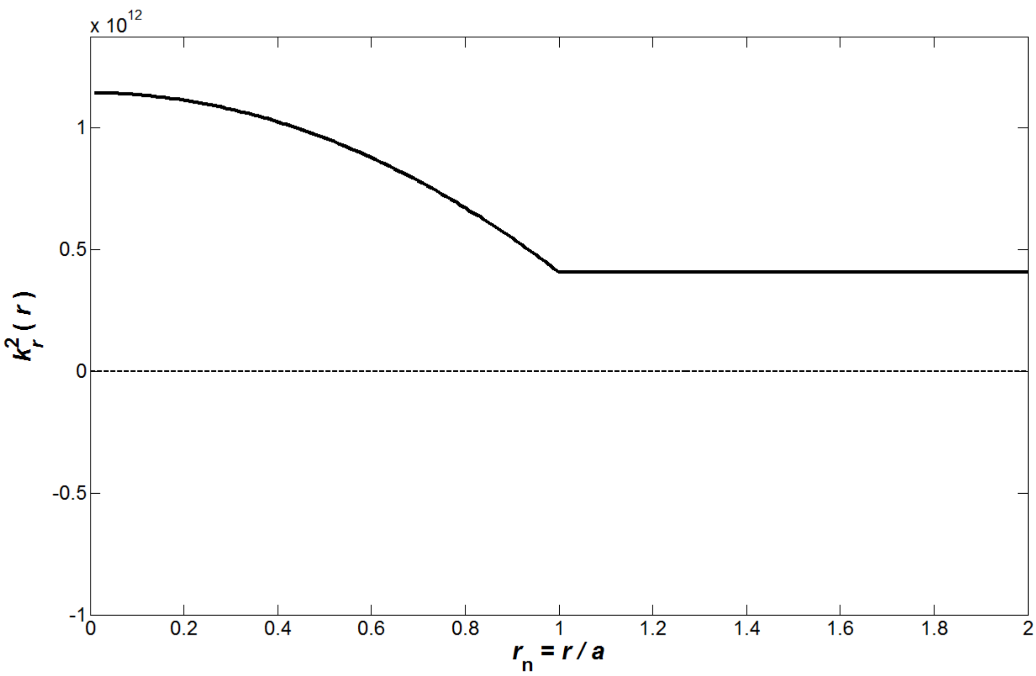

Fig. 2.3 A sample of Refracting Meridional Ray at $m=4$ and $\nu=0$ with $\beta=5.9857\times10^6$.

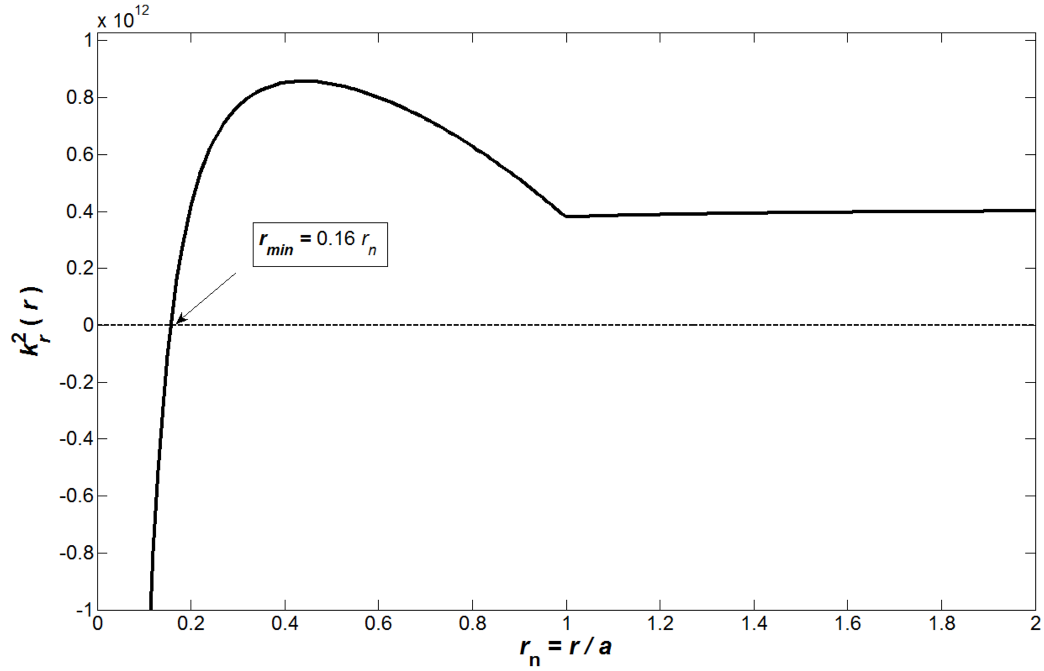

Fig. 2.4 A sample of Refracting Skew Ray at $m = 3$ and $\nu = 2$ with $\beta = 5.9857 \times 10^6$.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) PWE is an approximation to Helmholtz equation : True, PWE assumes that $\frac{\partial^2 u(\mathbf{r})}{\partial z^2} \to 0$, hence this term is present in Helmholtz equation, but it is missing in paraxial wave equation (PWE).

b) The intensity profile of a Gaussian beam changes with the radial parameter : True as illustrated in Fig. 1.1 of lecture notes entitled, "ECE 635_Free space propagation notes_Eylul 2011_HTE".

c) The source field expression of a Bessel Gaussian beam contains Bessel function and Laguerre function : False, it only contains Bessel function, Gaussian exponential and the phase term as indicated in (1.2) and (1.3).

d) PWE test can only be applied to the beam after it has propagated for some distance : True, for $\frac{\partial^2 u(\mathbf{r})}{\partial z^2} \to 0$ to be valid, beam must have propagated a certain distance.

e) The numerical aperture of a graded index fibre increases with the radial parameter : False, according to Fig. 2. 13 of lecture notes entitled, "ECE 635_Notes on Propagation in GI fibres_2013_HTE".

f) Bessel and modified Bessel Gaussian beams have similar intensity profiles : False, by comparing Figs. 1.3 and 1.4 of lecture notes entitled, "ECE 635_Free space propagation notes_Eylul 2011_HTE".