Çankaya University – ECE Department – ECE 635 (MT)

Student Name : Date : 22.11.2011

Student Number : Open book exam

Questions

1. (70 Points) On source plane, having cylindrical coordinates of s, ϕ_s, Sinusoidal / Hyperbolic Gaussian Beam is given by

$$u_s\ s,\phi_s\ = \sum_{\ell=1}^{2} A_\ell \exp\left[-k\alpha_\ell s^2 + \sin\phi_s + \cos\phi_s\ D_{s\ell} s\right] \qquad (1)$$

where A_ℓ is the amplitude coefficient, $k = 2\pi/\lambda$ is the wave number with λ being the wavelength, $\alpha = 1/(k\alpha_s^2) + 0.5\,j/F_s$ where α_s and F_s respectively refer to radial Gaussian source size and focusing parameter, D is the displacement parameter.

On the receiver plane having cylindrical coordinates of r, ϕ_r, z, the field of the same beam will be given by

$$u_r\ r,\phi_r,z\ = \sum_{\ell=1}^{2} \frac{A_\ell}{1+2\,j\alpha_\ell z} \exp\left[-\frac{k\alpha_\ell r^2 - \cos\phi_r + \sin\phi_r\ rD_{s\ell} - jk^{-1}D_{s\ell}^2 z}{1+2\,j\alpha_\ell z}\right] \qquad (2)$$

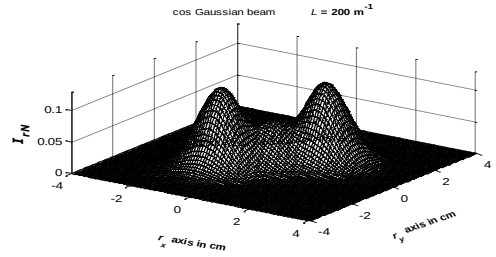
where, z is the propagation distance and $j = \sqrt{-1}$.

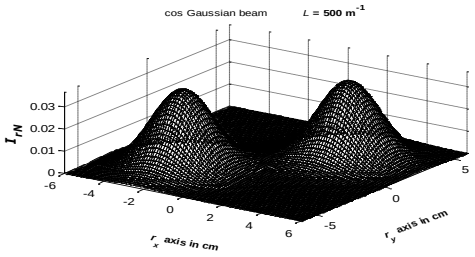
Insert Eq. (2) into the following paraxial wave equation and show that it is satisfied either by hand derivation or by MATLAB code. Write your MATLAB code here.

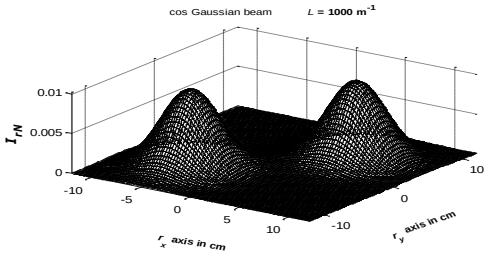
$$\text{PWE}: \frac{\partial^2 u_r\ r,\phi_r,z}{\partial r^2} + \frac{1}{r}\frac{\partial u_r\ r,\phi_r,z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_r\ r,\phi_r,z}{\partial^2 \phi_r} + 2\,jk\frac{\partial u_r\ r,\phi_r,z}{\partial z} = 0 \qquad (3)$$

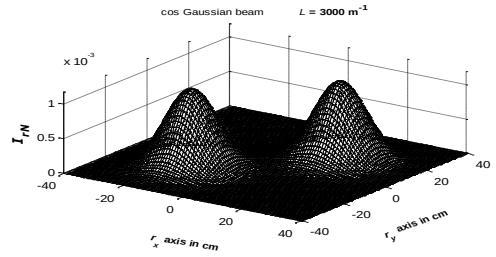
By setting the source parameters to $|A_\ell| = 1$, $\alpha_s = 1$ cm, $F_s \to \infty$, $\lambda = 1.55$ μm, $D = 200$ m , plot in MATLAB, the receiver intensity profiles at $z = 200$ m, 500 m, 1 km, 3 km for cos, sine, cosh and sinh Gaussian beams. Send these figure files (copied into one single file) to h.eyyuboglu@cankaya.edu.tr address.

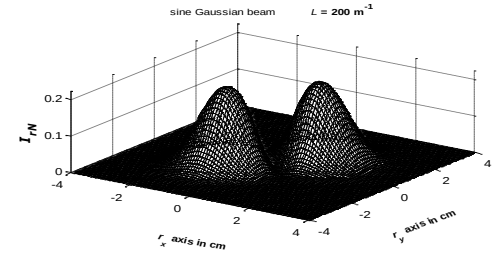
Solution : For the derivation of $u_r\ r,\phi_r,z$, see Notes for ECE 635_Eylul 2011 available on the course web page. To see that paraxial wave equation is satisfied, use PWETest.m. The relevant plots are shown below.

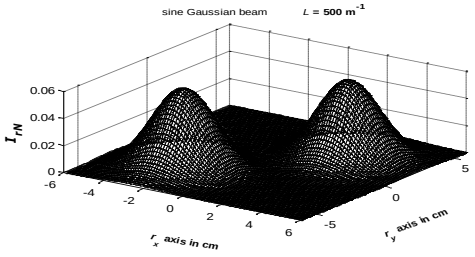

cos Gaussian beam L = 200 m-1
I rN
0.1
0.05
0
-4
-2
0
2
4
r x axis in cm
r y axis in cm

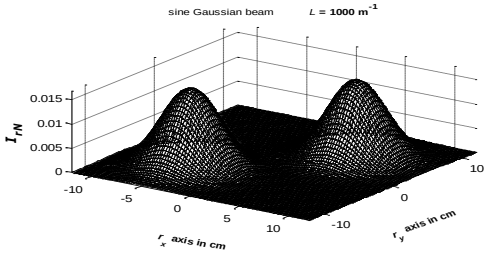

cos Gaussian beam L = 500 m-1
I rN
0.03
0.02
0.01
0
-6
-4
-2
0
2
4
6
5
0
-5
r x axis in cm
r y axis in cm

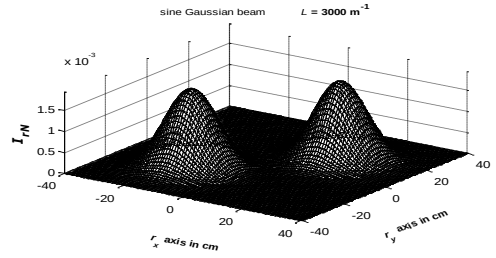

cos Gaussian beam L = 1000 m-1
I rN
0.01
0.005
0
-10
-5
0
5
10
r x axis in cm
r y axis in cm

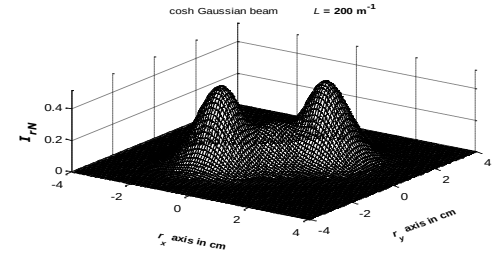

cos Gaussian beam L = 3000 m-1
x 10-3
I rN
1
0.5
0
-40
-20
0
20
40
r x axis in cm
r y axis in cm

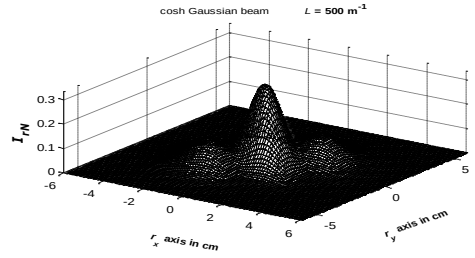

sine Gaussian beam L = 200 m-1
I rN
0.2
0.1
0
-4
-2
0
2
4
r x axis in cm
r y axis in cm

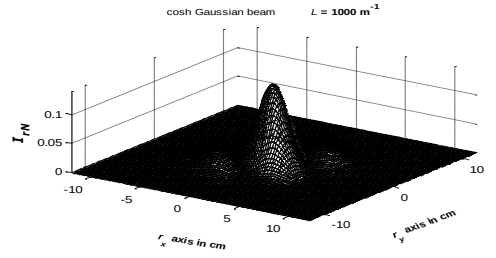

sine Gaussian beam L = 500 m-1
I rN
0.06
0.04
0.02
0
-6
-4
-2
0
2
4
6
5
0
-5
r x axis in cm
r y axis in cm

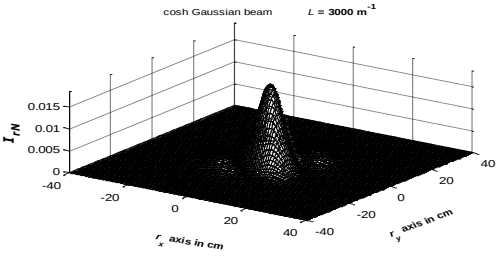

sine Gaussian beam L = 1000 m-1
I rN
0.015
0.01
0.005
0
-10
-5
0
5
10
r x axis in cm
r y axis in cm

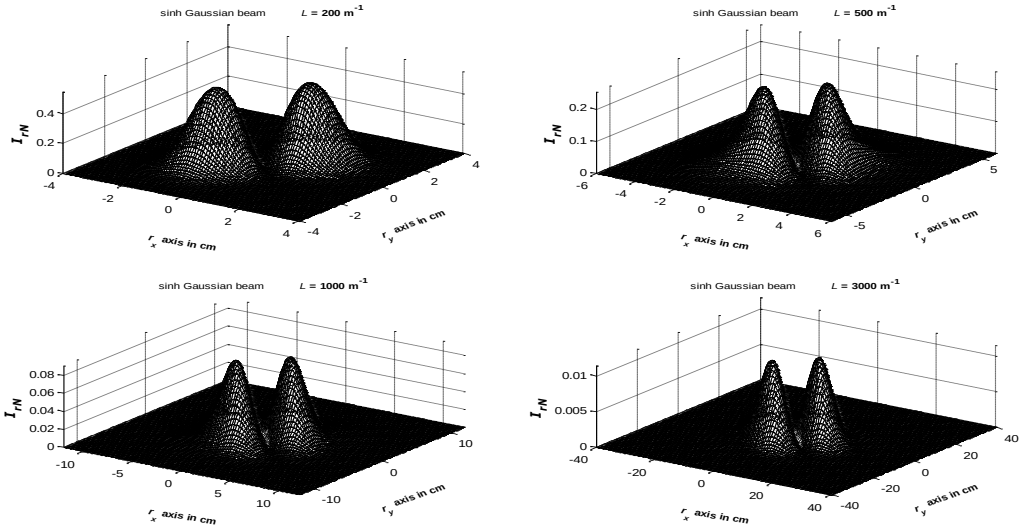

sine Gaussian beam L = 3000 m-1
x 10-3
I rN
1.5
1
0.5
0
-40
-20
0
20
40
r x axis in cm
r y axis in cm


cosh Gaussian beam L = 200 m-1
I rN
0.4
0.2
0
-4
-2
0
2
4
r x axis in cm
r y axis in cm


cosh Gaussian beam L = 500 m-1
I rN
0.3
0.2
0.1
0
-6
-4
-2
0
2
4
6
5
0
-5
r x axis in cm
r y axis in cm


cosh Gaussian beam L = 1000 m-1
I rN
0.1
0.05
0
-10
-5
0
5
10
r x axis in cm
r y axis in cm


cosh Gaussian beam L = 3000 m-1
I rN
0.015
0.01
0.005
0
-40
-20
0
20
40
r x axis in cm
r y axis in cm


sinh Gaussian beam L = 200 m-1
sinh Gaussian beam L = 500 m-1
sinh Gaussian beam L = 1000 m-1
sinh Gaussian beam L = 3000 m-1
I rN
r x axis in cm
r y axis in cm

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) Plane wave has flat phase front : True, the phase front (i.e. radius of curvature) of plane wave is infinity

b) Spherical wave has curved phase front : True, but at large distances, it is treated as flat wave front.

c) Propagation in graded index fibres takes place in the form of helical rays : Particularly true for skew rays.

d) Fermat principle is based on the propagation of rays in a medium where the refractive index changes : Not necessarily, Fermat principle also applies to mediums where refractive index remains (spatially) constant.

e) Paraxial wave equation is an approximation and applies to propagations confined around one axis : True, paraxial wave equation assumes that propagation is mainly confined around on-axis, or the changes on the transverse plane are much smaller with respect to longitudinal changes.

f) Plane wave has no phase variation when propagating : False, the phase of plane wave varies as (in free space) kz in propagation.