Çankaya University – ECE Department – ECE 635 (MT)

Student Name : Date : 19.11.2013

Student Number : Open book exam

Questions

1. (35 Points) A fibre with $n_1 = 1.48,\ n_2 = 1.47,\ a = 4\ \mu\text{m}$ is operated at a wavelength of $\lambda = 1.55\ \mu\text{m}$. Determine the number of modes that will propagate in this fibre. Calculate the propagation constant for each mode. Verify that these modes will actually propagate in the fibre. For evaluations, you can use the m file Findingbeta_Exp0.m, available on the course webpage, ece635.cankaya.edu.tr.

Solution : First, we find

$$V = \ ak\left(n_1^2 - n_2^2\right)^{0.5} = 2.785 \tag{1.1}$$

This means that the modes that have zero crossing curves that are not contained within the $V =\ 2.785$ will not propagate in this fibre. By using Findingbeta_Exp0.m file and inserting the given parameters of the fibre, we see that only TM_{01}, TE_{01} and HE_{11} modes will propagate in this fibre. Their associated plots are given in Figs. 1.1, 1.2 and 1.3.

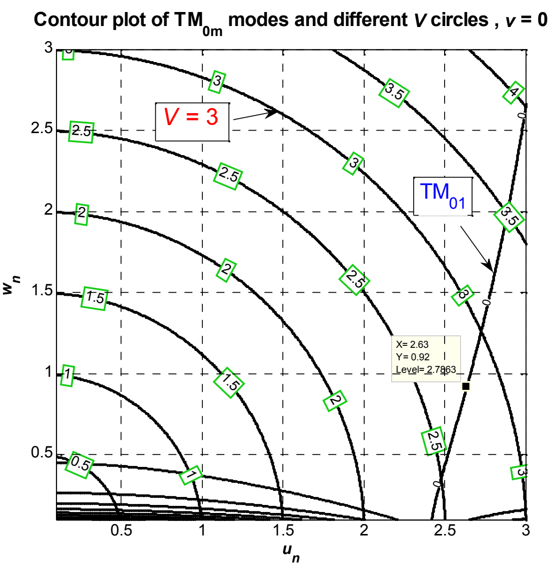

Fig. 1.1 Contour plot for the zero crossing curve of TM_{01} mode and various V circles.

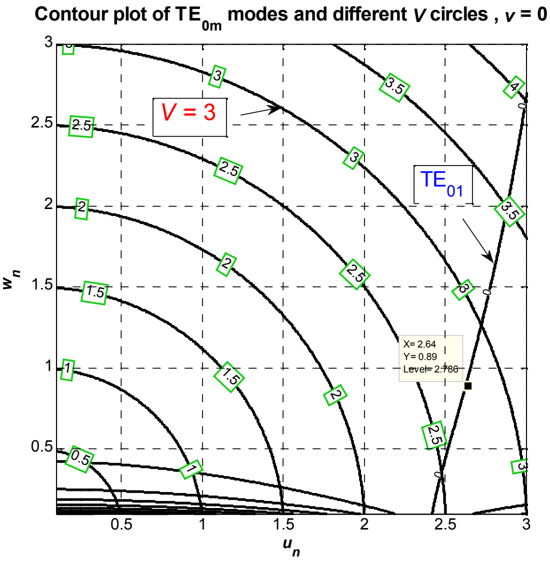

Fig. 1.2 Contour plot for the zero crossing curve of TE_{01} mode and various V circles.

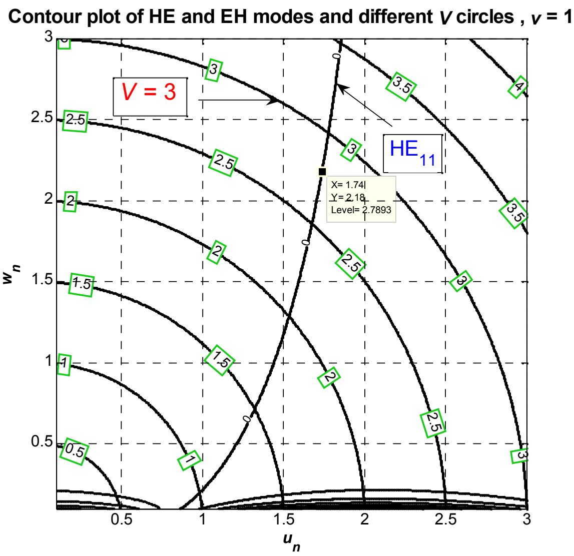

Fig. 1.3 Contour plot for the zero crossing curve of HE_{11} mode and various V circles.

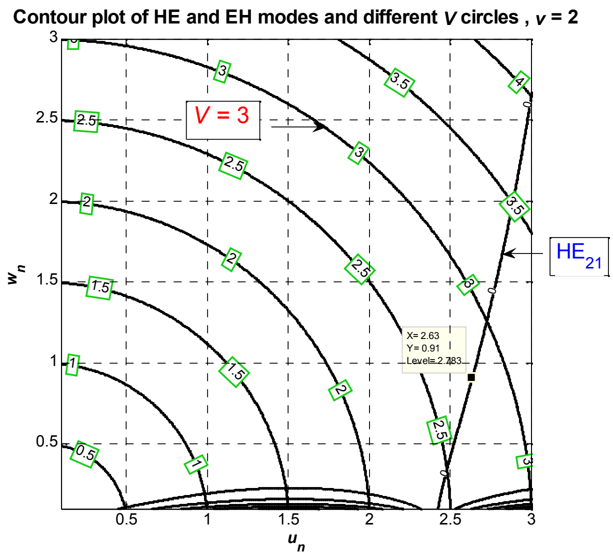

Fig. 1.4 Contour plot for the zero crossing curve of HE_{21} mode and various V circles.

From u_n and w_n readings indicated with the marking on Figs. 1.1, 1.2, 1.3 and 1.4, we evaluate the propagation constants for these modes as follows

$$k_1 = 5.9994\times10^6 \quad , \quad k_2 = 5.9584\times10^6$$

$$\mathrm{TM}_{01} \;:\; u_n = 2.63 \;,\; w_n = 0.92$$

$$\beta_n = \left(a^2k_1^2 - u_n^2\right)^{0.5} = 23.8532 \;\text{ or }\; \beta_n = \left(a^2k_2^2 + w_n^2\right)^{0.5} = 23.8533$$

$$\beta = \frac{\beta_n}{a} = 5.9633\times10^6$$

$$\mathrm{TE}_{01} \;:\; u_n = 2.64 \;,\; w_n = 0.89$$

$$\beta_n = \left(a^2k_1^2 - u_n^2\right)^{0.5} = 23.8521 \;\text{ or }\; \beta_n = \left(a^2k_2^2 + w_n^2\right)^{0.5} = 23.8522$$

$$\beta = \frac{\beta_n}{a} = 5.963\times10^6$$

$$\mathrm{HE}_{11} \;:\; u_n = 1.74 \;,\; w_n = 2.18$$

$$\beta_n = \left(a^2k_1^2 - u_n^2\right)^{0.5} = 23.9346 \;\text{ or }\; \beta_n = \left(a^2k_2^2 + w_n^2\right)^{0.5} = 23.9351$$

$$\beta = \frac{\beta_n}{a} = 5.9838\times10^6$$

$$\mathrm{HE}_{21} \;:\; u_n = 2.63 \;,\; w_n = 0.91$$

$$\beta_n = \left(a^2k_1^2 - u_n^2\right)^{0.5} = 23.8532 \;\text{ or }\; \beta_n = \left(a^2k_2^2 + w_n^2\right)^{0.5} = 23.8529$$

$$\beta = \frac{\beta_n}{a} = 5.9632\times10^6 \qquad (1.2)$$

As seen, all propagation constant values satisfy

$$k_2 = n_2 k \le \beta \le k_1 = n_1 k \tag{1.3}$$

2. (35 Points) Quadratic graded index fibre with $n_1 = 1.50,\ n_2 = 1.48,\ a = 15\ \mu\text{m}$ is given. List at least three possible sets of x_0, y_0, θ_{x0} and θ_{y0} for the following ray categories

A. Propagating Meridional Rays
B. Propagating Skew Rays
C. Refracting Meridional Rays
D. Refracting Skew Rays

By using , the m file Ray_tracing_GI_Exp1.m, available on the course webpage, ece635.cankaya.edu.tr, make plots of the rays with different x_0, y_0, θ_{x0} and θ_{y0} settings verifying the category of the ray.

Solution : From (2.30) of the notes entitled, "ECE 635_Notes on Propagation in GI fibres_2013_HTE", we know that for meridional rays

$$x_0\theta_{y0} = y_0\theta_{x0} \tag{2.1}$$

Thus for skew rays

$$x_0\theta_{y0} \neq y_0\theta_{x0} \tag{2.2}$$

By arranging the parameters suitably, we get (one sample plot is provided for each case)

A. Propagating Meridional Rays

$$\begin{aligned}
&1)\ x_0 = 0.2a,\ y_0 = 0.09a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.09,\ r_{\max} = 12.52\ \mu\text{m} \\
&2)\ x_0 = 0.2a,\ y_0 = 0.06a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.06,\ r_{\max} = 10.854\ \mu\text{m} \\
&3)\ x_0 = 0.15a,\ y_0 = 0.05a,\ \theta_{x0} = 0.15,\ \theta_{y0} = 0.05,\ r_{\max} = 14.72\ \mu\text{m}
\end{aligned} \tag{2.3}$$

B. Propagating Skew Rays

$$\begin{aligned}
&1)\ x_0 = 0.1a,\ y_0 = 0.04a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.07,\ r_{\max} = 11.294\ \mu\text{m} \\
&2)\ x_0 = 0.05a,\ y_0 = 0.02a,\ \theta_{x0} = 0.01,\ \theta_{y0} = 0.01,\ r_{\max} = 1.5048\ \mu\text{m} \\
&3)\ x_0 = 0.2a,\ y_0 = 0.01a,\ \theta_{x0} = 0.03,\ \theta_{y0} = 0.03,\ r_{\max} = 4.617\ \mu\text{m}
\end{aligned} \tag{2.4}$$

C. Refracting Meridional Rays

$$\begin{aligned}
&1)\ x_0 = 0.2a,\ y_0 = 0.09a,\ \theta_{x0} = 0.2,\ \theta_{y0} = 0.09,\ r_{\max} = 20.412\ \mu\text{m} \\
&2)\ x_0 = 0.2a,\ y_0 = 0.06a,\ \theta_{x0} = 0.2,\ \theta_{y0} = 0.06,\ r_{\max} = 19.434\ \mu\text{m} \\
&3)\ x_0 = 0.2a,\ y_0 = 0.05a,\ \theta_{x0} = 0.2,\ \theta_{y0} = 0.05,\ r_{\max} = 19.193\ \mu\text{m}
\end{aligned} \tag{2.5}$$

D. Refracting Skew Rays

$$\begin{aligned} &1)\ x_0 = 0.5a,\ y_0 = 0.08a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.15,\ r_{\max} = 17.432\ \mu\text{m} \\ &2)\ x_0 = 0.5a,\ y_0 = 0.1a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.14,\ r_{\max} = 16.858\ \mu\text{m} \\ &3)\ x_0 = 0.7a,\ y_0 = 0.1a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.12,\ r_{\max} = 16.776\ \mu\text{m} \end{aligned} \tag{2.5}$$

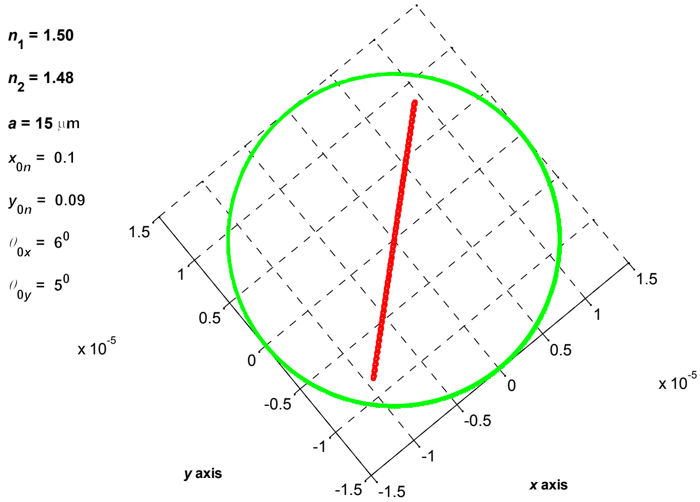

Fig. 2.1 Propagating meridional ray projection for the settings in A.1).

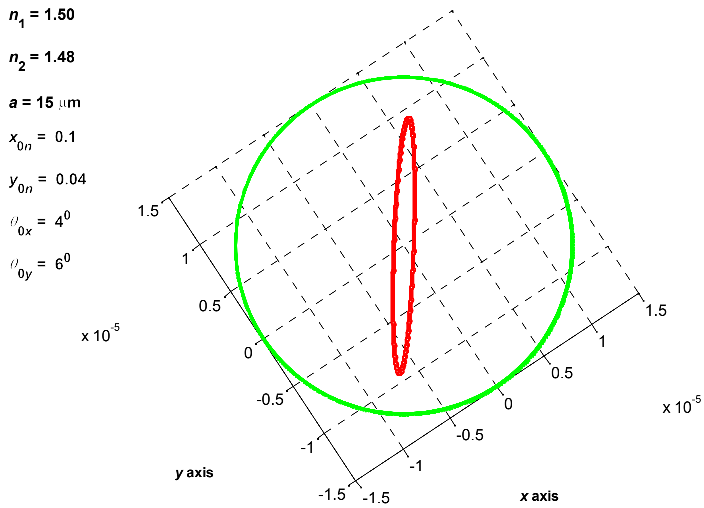

Fig. 2.2 Propagating skew ray projection for the settings in B.1).

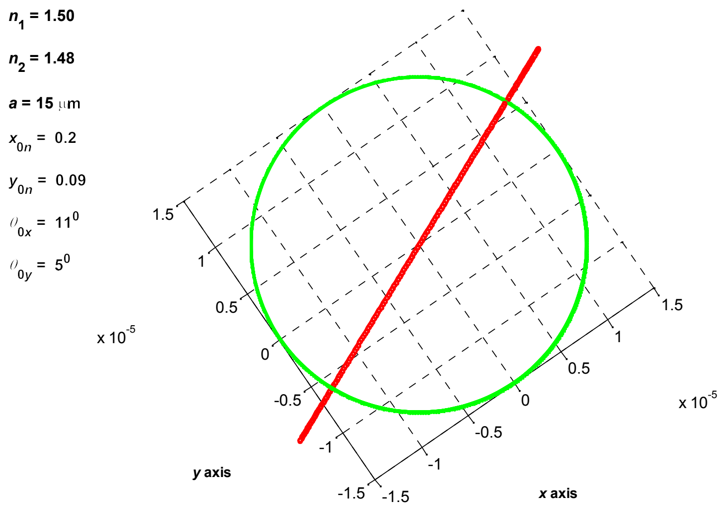

Fig. 2.3 Refracting meridional ray projection for the settings in C.1).

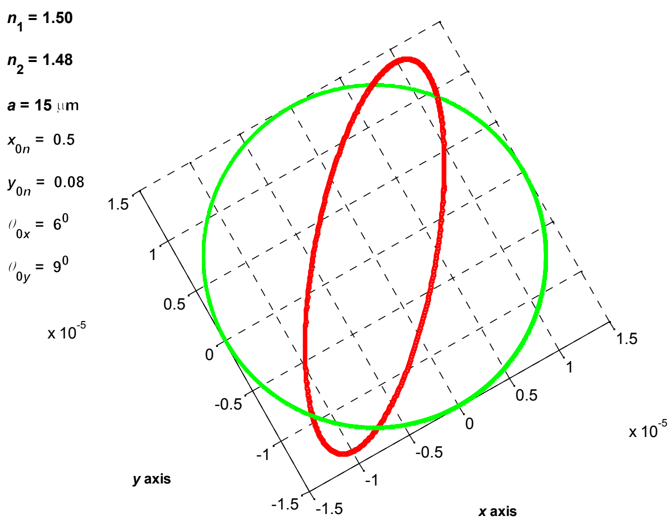

Fig. 2.4 Refracting skew ray projection for the settings in D.1).

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) In graded index fibre, the ray path of a meridional ray is a straight line : False, as illustrated in Fig. 1.2 of notes entitled, "ECE 635_Notes on Propagation in GI fibres_2013_HTE".

b) Single mode fibre has a narrow acceptance cone (numerical aperture) : True since numerical aperture (NA) is defined as $\mathrm{NA} = \left(n_1^2 - n_2^2\right)^{0.5}$.

c) The cladding radius of a single mode fibre is the same as the multimode step index fibre : Not exactly, in single mode fibre, the core radius is much smaller, but the cladding radius is the same as the multimode step index fibre. So the cladding region is wider in single mode fibre. On the other hand, in single mode fibre, appreciable portion of HE_{11} mode propagates in cladding. Therefore in single mode fibres, more attention is paid to the quality of cladding material.

d) As the wavelength of operation is increased, the normalized frequency of the fibre increases as well : False since $V^2 = a^2 k^2 \left(n_1^2 - n_2^2\right) = a^2 \dfrac{4\pi^2}{\lambda^2}\left(n_1^2 - n_2^2\right)$

e) The numerical aperture of multimode step index fibre is greater than the (multimode) graded index fibre : True, according to Figs. 2.13, 2.14, 2.15 in notes entitled, "“ECE 635_Notes on Propagation in GI fibres_2013_HTE".

f) Many modes propagate in fibres : It should say, many modes propagate in multimode (step and graded index) fibres.