Çankaya University – ECE Department – ECE 635 (FE)

Student Name : Date : 14.01.2014

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Questions

1. (70 Points) The expression in (1.1) is for the mutual coherence function of partially coherent sinusoidal hyperbolic Gaussian beam.

Explain and describe the followings

a. The significance of using the coordinates with different indices, i.e., r_{1x}, r_{2x} and r_{1y}, r_{2y} and its relation to intensity.

b. The significance and the role of partial coherence parameter, σ_s . The significance and the role of displacement parameters, D_x, D_y .

c. The different beams that can be obtained from the expression in (1.1).

Obtain a fully coherent Gaussian beam intensity expression by setting, $N = 1,\ \sigma_s \to \infty,\ D_x = D_y = 0$ in (1.1) and show the intermediate expressions.

Use ParCoh_SinoHypR_DersS.m available on the course webpage with the present settings and alternately change ib on line 29 to $1,\ 2,\ 3,\ 4$. Identify the resulting plots in terms of beam types by running ParCoh_SinoHypR_DersS.m, also for each beam describe the beam evolution at propagation distances, $L = 50$ m, 500 m, 1 km, 5 km .

$$
\begin{aligned}
&\Gamma_r\left(r_{1x}, r_{1y}, r_{2x}, r_{2y}, L\right) = k\sigma_s^2 \exp\left[j\frac{k}{2L}\left(r_{1x}^2 - r_{2x}^2 + r_{1y}^2 - r_{2y}^2\right)\right] \sum_{\ell_1=1}^{N}\sum_{\ell_2=1}^{N} \\
&\frac{A_{\ell_1} A_{\ell_1}^*}{\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{x\ell_1} + \alpha_{x\ell_2}^*\right)L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]^{0.5} \left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{y\ell_1} + \alpha_{y\ell_2}^*\right)L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]^{0.5}} \\
&\exp\left(\frac{0.5}{L\left(k\alpha_{x\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)}\left\{\left(-jkr_{1x} + D_{x\ell_1}L\right)^2\sigma_s^2 + \frac{\left[-j\left(r_{1x} - r_{2x}\right)kL + j\left(\alpha_{x\ell_1}L - j\right)k^2 r_{2x}\sigma_s^2 + D_{x\ell_1}L^2 + \left(k\alpha_{x\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)D_{x\ell_2}^* L\right]^2}{k\left[k\alpha_{x\ell_1}\alpha_{x\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{x\ell_1} + \alpha_{x\ell_2}^*\right)L^2 + j\left(\alpha_{x\ell_1} - \alpha_{x\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]}\right\}\right) \\
&\exp\left(\frac{0.5}{L\left(k\alpha_{y\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)}\left\{\left(-jkr_{1y} + D_{y\ell_1}L\right)^2\sigma_s^2 + \frac{\left[-j\left(r_{1y} - r_{2y}\right)kL + j\left(\alpha_{y\ell_1}L - j\right)k^2 r_{2y}\sigma_s^2 + D_{y\ell_1}L^2 + \left(k\alpha_{y\ell_1}\sigma_s^2 L + L - jk\sigma_s^2\right)D_{y\ell_2}^* L\right]^2}{k\left[k\alpha_{y\ell_1}\alpha_{y\ell_2}^*\sigma_s^2 L^2 + \left(\alpha_{y\ell_1} + \alpha_{y\ell_2}^*\right)L^2 + j\left(\alpha_{y\ell_1} - \alpha_{y\ell_2}^*\right)k\sigma_s^2 L + k\sigma_s^2\right]}\right\}\right)
\end{aligned}
\tag{1.1}
$$

Solution : a. The coordinates with different indices measure the correlation of point one of the receiver plane with respect to another one. This way if they are selected to be coincident , i.e., $r_{1x} = r_{2x}$ and $r_{1y} = r_{2y}$, then mutual coherence function become the intensity, that is

$$I_r\left(r_x, r_y\right) = \Gamma_r\left(r_{1x} = r_{2x} = r_x, r_{1y} = r_{2y} = r_y, L\right)$$

b. The partial coherence parameter indicates the phase discontinuities or phase jumps in the sinusoidal changes of the optical wave along time or spatial axis. Such an event can only be detected by taking two points, in time or space, partial coherence can only be expressed by mutual coherence function. Besides phase changes are random, thus σ_s^2 is the variance in the probability function of these changes. The displacement parameters are used to generate sine, cos, sinh and cosh beams. They are set to zero for fundamental Gaussian and annular Gaussian beams.

c. Since the derivation of (1.1) or (5.17) in notes, entitled, "ECE 635_Free space propagation notes_Eylul 2011_HTE" is based on (5.7) of the same notes, which can deliver all sinusoidal hyperbolic Gaussian beams, it can safely be said that from (1.1), by setting $A_\ell,\ \alpha_{x\ell},\ \alpha_{y\ell},\ D_{x\ell},\ D_{y\ell}$ appropriately we obtain fundamental Gaussian, annular Gaussian sine, cos, sinh, cosh Gaussian beams.

The reduction of (1.1) to a fully coherent Gaussian beam is carried out in ECE 635_Free space propagation Exercise51_Jan 2014_HTE, so the result is

$$I_r\left(r_x,r_y\right)=\frac{|A|^2}{\left[|\alpha_x|^2L^2+j\left(\alpha_x-\alpha_x^*\right)L+1\right]^{0.5}\left[|\alpha_y|^2L^2+j\left(\alpha_y-\alpha_y^*\right)L+1\right]^{0.5}}\exp\left[\frac{-0.5k\left(\alpha_x+\alpha_x^*\right)r_x^2}{|\alpha_x|^2L^2+j\left(\alpha_x-\alpha_x^*\right)L+1}\right]\exp\left[\frac{-0.5k\left(\alpha_y+\alpha_y^*\right)r_y^2}{|\alpha_y|^2L^2+j\left(\alpha_y-\alpha_y^*\right)L+1}\right] \tag{1.2}$$

The observations regarding beam evolutions at $L=50\text{ m},\ 500\text{ m},\ 1\text{ km},\ 5\text{ km}$: Diffraction is common to all beams, hence beam foot print becomes larger and larger with increasing propagation distance. Particular details are ;

a) Gaussian beam : This beam keeps the Gaussian profile at all propagation instances.

b) Annular Gaussian beam : Initially, the central portion is hollow, but with increasing propagation distance a beak rises in the centre and a surrounding ring appears around it.

c) Sine Gaussian beam : Starts as two separate lobes aligned around slanted axis, upon propagation turns into sinh Gaussian beam.

d) Cos Gaussian beam : Starts with a main lobe in the centre and partial rings on the other edges, then turns into a cosh Gaussian beam.

e) Cosh Gaussian beam : Starts with two main decentred connected lobes, eventually turns into cos Gaussian beam.

f) Sinh Gaussian beam : Starts with two main decentred unconnected lobes, eventually turns into sine Gaussian beam.

To understand how different beams are formed, we offer the following table that contain the settings for $A_\ell,\ \alpha_{x\ell},\ \alpha_{y\ell},\ D_{x\ell},\ D_{y\ell}$

	Beam name					
Parameter	Gaussian	Annular Gaussian	Sine Gaussian	Cos Gaussian	Cosh Gaussian	Sinh Gaussian
A_1	1	1	j	1	1	1
A_2	0	≥ -1	-j	1	1	-1
D_{x_1}, D_{y_1}	0	0	>0 imaginary	>0 imaginary	>0 real	>0 real
D_{x_2}, D_{y_2}	0	0	<0 imaginary	<0 imaginary	<0 real	<0 real
α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}
α_{sx_2}, α_{sy_2}		$<\alpha_{sx_1}$, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}	α_{sx_1}, α_{sy_1}

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) ABCD matrix representation can be used instead of the Huygens Fresnel integral to find the receiver plane field : True, ABCD matrix representation is particularly useful, when there are optical elements on the propagation path.

b) Partial coherence is related to phase changes : True, such phase changes occur both along time and spatial axis. In the mutual coherence function representation, we use spatial axes.

c) From source (transmitter) plane, when we launch one type of beam, on receiver plane we obtain a different type of beam : True, particularly for sinusoidal hypergeometric Gaussian beams.

d) As the propagation distance is increased, beam footprint (beam area) on receiver plane decreases : False (unless focusing parameter is used intentionally), just the reverse occurs and beam footprint always beams larger with increasing propagation distance.

e) Bessel Gaussian beam is obtained by multiplying the Gaussian exponential by Bessel function of second kind : False as defined in (1.3) of ECE 635_Free space propagation notes_Eylul 2011_HTE, this is modified Bessel Gaussian beam.