# Çankaya University – ECE Department – ECE 474 (MT)

Student Name :
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Open book exam, Duration : 2 Hours

Questions

1. (40 Points) A parabolic (quadratic) graded index fibre has $n_1 = 1.50$, $n_2 = 1.48$, $a = 10\ \mu\text{m}$. Using the Fermat principle and the related differential equations, find the range and the limits of propagating rays in this fibre in terms of initial launching conditions, i.e. $x_0$, $y_0 =$, $\theta_{x0}$ and $\theta_{y0}$. In the set of parameters, $x_0$, $y_0 =$, $\theta_{x0}$ and $\theta_{y0}$, identify which set of $x_0$, $y_0 =$, $\theta_{x0}$ and $\theta_{y0}$ will give

   a) Propagating Meridional Rays
   b) Propagating Skew Ray
   c) Refracting Rays

   For at least two rays in category a) and b), find $r_{\min}$, $r_{\max}$ and $z_p$.

Solution: For classification of rays according to initial launching parameters, we use "Notes on ray propagation in GI fibre-2703211.pdf"

To this end, we take the formulation for $z_p$

$$z_p = \frac{a}{2\sqrt{2\Delta}} \tan^{-1}\left[\frac{2a\sqrt{2\Delta}\,(x_0\theta_{x0} + y_0\theta_{y0})}{2\Delta(x_0^2 + y_0^2) - a^2(\theta_{x0}^2 + \theta_{y0}^2)}\right]$$

Assuming this $z_p$ refers to $z_p$ of $r_{min}$, then $z_{pmin} = z_p$ (of above) and $z_{pmax}$ which will correspond to $\rho_{max}$ will be

$$z_{pmax} = \frac{a}{2\sqrt{2\Delta}} \left\{ \tan^{-1}\left[ \quad \right] + \pi \right\}$$

Condition for meridional ray will be

$$\rho_{min} = \left[ x^2(z_{pmin}) + y^2(z_{pmin}) \right]^{1/2} = 0$$

When $z_{pmin}$ from above is substituted in the expression of $\rho_{min}$, after rearrangement, we get the simple relationship that

Any $x_0, \theta_{x0}, y_0, \theta_{y0}$ satisfying $x_0 \theta_{y0} = y_0 \theta_{x0}$ will be meridional rays (propagating and refracting)

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Conditions for refraction will be

$$r_{max} = \left[ x^2(z_{pmax}) + y^2(z_{pmax}) \right]^{1/2} \geq a$$

Unfortunately in this case, it not so easy to acheive a simple relationship like in the case of $r_{min} = 0$. One difficulty is we must be sure about $z_{pmax}$, since it is not clear which of the two $z_p$ expressions above will give $z_{pmin}$ and $z_{pmax}$.

Here a better way is to use the MATLAB experiment m code, Ray-tracing-GL-Exp2.m.

a) Using $x_0 \theta_{y0} = y_0 \theta_{x0}$ for propagating Meridional Rays we may have

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1) $x_0 = 0.05a$ , $\theta_{y0} = 0.03$ , $y_0 = 0.03a$ , $\theta_{x0} = 0.05$

Here $r_{min} \simeq 0$, $r_{max} = 3.6\ \mu m$ , $z_{pmax} = 86.28\ \mu m$

2) $x_0 = 0.07a$ , $\theta_{y0} = 0.09$ , $y_0 = 0.09a$ , $\theta_{x0} = 0.07$

Here $r_{min} \simeq 0$ , $r_{max} = 7.07\ \mu m$ , $z_{pmax} = 86.28\ \mu m$

b) For propagating skew rays, we take

1) $x_0 = 0.05a$ , $y_0 = 0.03a$ , $\theta_{x0} = 0.03$ , $\theta_{y0} = 0.05$

Here $r_{min} = 0.27\ \mu m$ , $r_{max} = 3.6\ \mu m$ , $z_p = 87.38\ \mu m$

2) $x_0 = 0.07a$ , $y_0 = 0.09a$ , $\theta_{x0} = 0.09$ , $\theta_{y0} = 0.07$

Here $r_{min} = 0.28\ \mu m$ , $r_{max} = 7.07\ \mu m$ , $z_{pmax} = 86.56\ \mu m$

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c) For Refracting Rays we take

1) $x_0 = 0.15a$, $\theta_{y0} = 0.09$, $y_0 = 0.09a$, $\theta_{x0} = 0.15$

Here $r_{min} \simeq 0$, $r_{max} = 10.85\ \mu m > a$

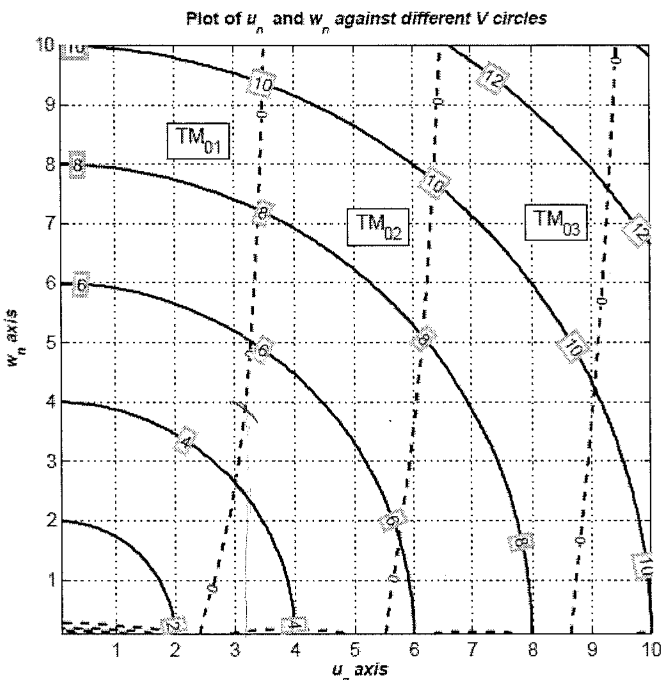
$z_p = 86.28\ \mu m$

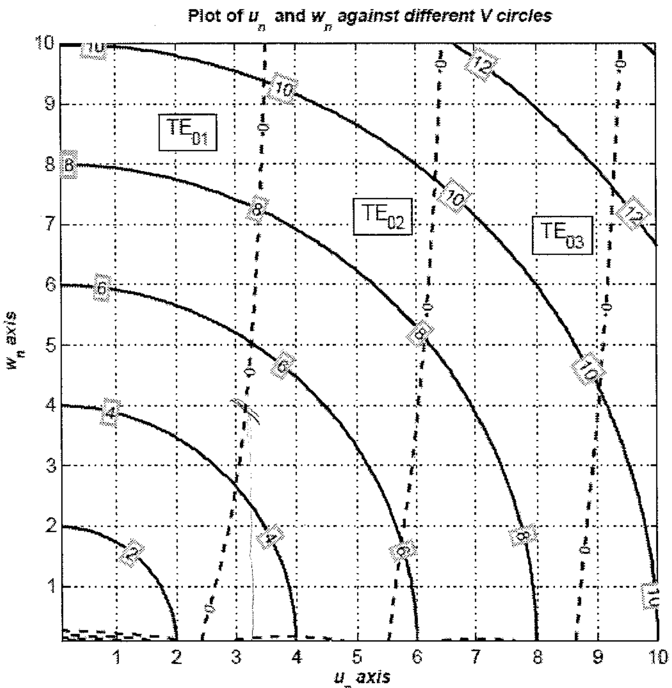
2) $x_0 = 0.15a$, $y_0 = 0.09a$, $\theta_{x0} = 0.09$, $\theta_{y0} = 0.15$

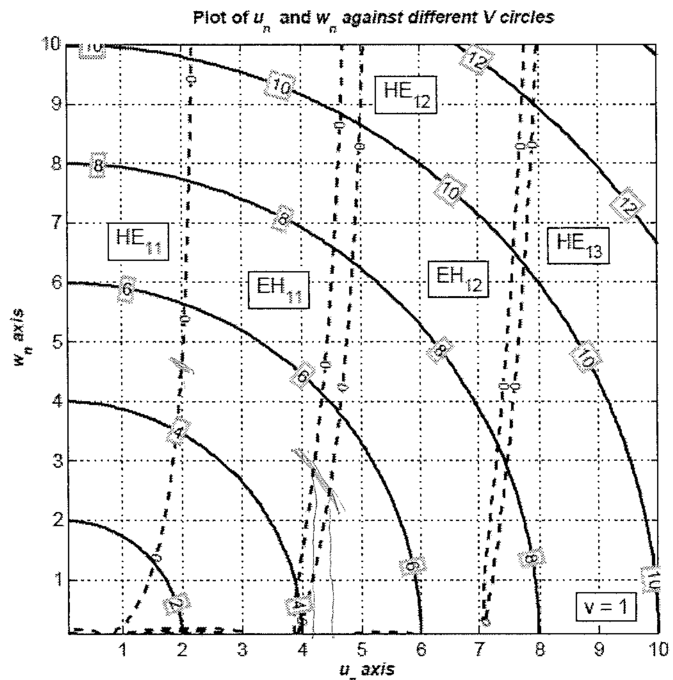
Here $r_{min} = 0.815\ \mu m$, $r_{max} = 10.82\ \mu m > a$

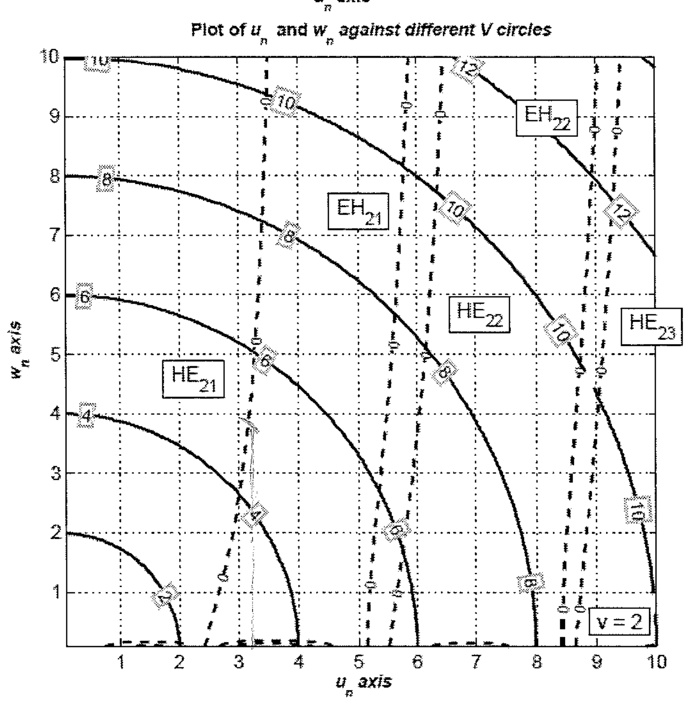
$z_p = 87.38\ \mu m$

2. (30 Points) For a fibre which has core radius of $a = 4\ \mu m$, refractive index difference of $\Delta = 0.0211$ and $n_1 = 1.50$, operating at $\lambda = 1.55\ \mu m$, find which modes will propagate from the following graphs. Evaluate numerically the propagation constant, $\beta$ of these modes. How can this fibre be made a single mode fibre ?, explain and make numeric evaluations.



Solution: From the given settings

$$\frac{n_1^2 - n_2^2}{n_1^2} = 2\Delta \ , \quad V = ak\left(n_1^2 - n_2^2\right)^{0.5}$$

$$= akn_1\left(2\Delta\right)^{0.5} = 4.996382 \simeq 5$$

From the given graphs of $TM_{0m}$, $TE_{0m}$, $EH_{\nu m}$, $EH_{\nu m}$ (for $\nu = 1$) and $HE_{\nu m}$, $EH_{\nu m}$ (for $\nu = 2$) we have the following propagating modes enclosed by the circle of $V = 5$.

For this we find the intersections of mode root (indicated by broken line) and $V = 5$ circle and reading off $W_n$ or $U_n$. Here we take $U_n$ and initially calculate

$$k_1 = n_1 k = 6.08051 \times 10^6$$

$$n_2 = \left(n_1^2 - 2n_1^2\Delta\right)^{1/2} = 1.468$$

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$k_2 = n_2 k = 5.951 \times 10^6$

For $TM_{01}$ $\beta = \left( \frac{w_n^2}{a^2} + k_2^2 \right)^{0.5} = \left( k_1^2 - \frac{u_n^2}{a^2} \right)^{0.5}$

Inserting $u_n = 3.15$, $\beta = 6.0293 \times 10^6$

For $TE_{01}$, $u_n = 3.2$, $\beta = 6.0276 \times 10^6$

For $HE_{11}$, $u_n = 2$, $\beta = 6.05991 \times 10^6$

For $EH_{11}$, $u_n = 4.2$, $\beta = 5.9889 \times 10^6$

For $HE_{12}$, $u_n = 4.5$, $\beta = 5.9755 \times 10^6$

For $HE_{21}$, $u_n = 3.2$, $\beta = 6.02765 \times 10^6$

This fibre can be made a single mode fibre by setting $V < 2.4$, which means eliminating all modes given above except $HE_{11}$ that has no cut-off value. As seen from the equation, $V = ak(n_1^2 - n_2^2)^{1/2}$, acheiving single mode condition can be established by three means

a) Lowering core radius, a

b) operating at higher wavelengths

c) Making core cladding index difference ($\Delta$) smaller

To give a numeric example, if we keep all other parameters constant and try to acheive single mode condition by lowering a

$V = 2.4 < ak(n_1^2 - n_2^2)^{0.5}$, then

$a < 1.9214\ \mu m$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In single mode fibre, the field is confined to the core : Mainly but if V is below 2, then the tail of the field extents well into the cladding

b) As wavelength increases, attenuation in fibre decreases : This is no such generalization, we know from the relevant graph that lowest loss window occurs at $\lambda = 1.55 \mu m$, next lowest is $\lambda = 1.31 \mu m$

c) In graded index fibres, ray path is straight lines : False, in graded index fibre, ray path is sinusoidal as shown in the notes

d) Dispersion arises from the time delay difference between rays or modes : True

this is exactly the soure of dispersion

e) Monochromatic optical source contains many wavelengths : False

Monochromatic optical source emit at a single wavelength. Polychromatic sources radiate at many wavelengths.